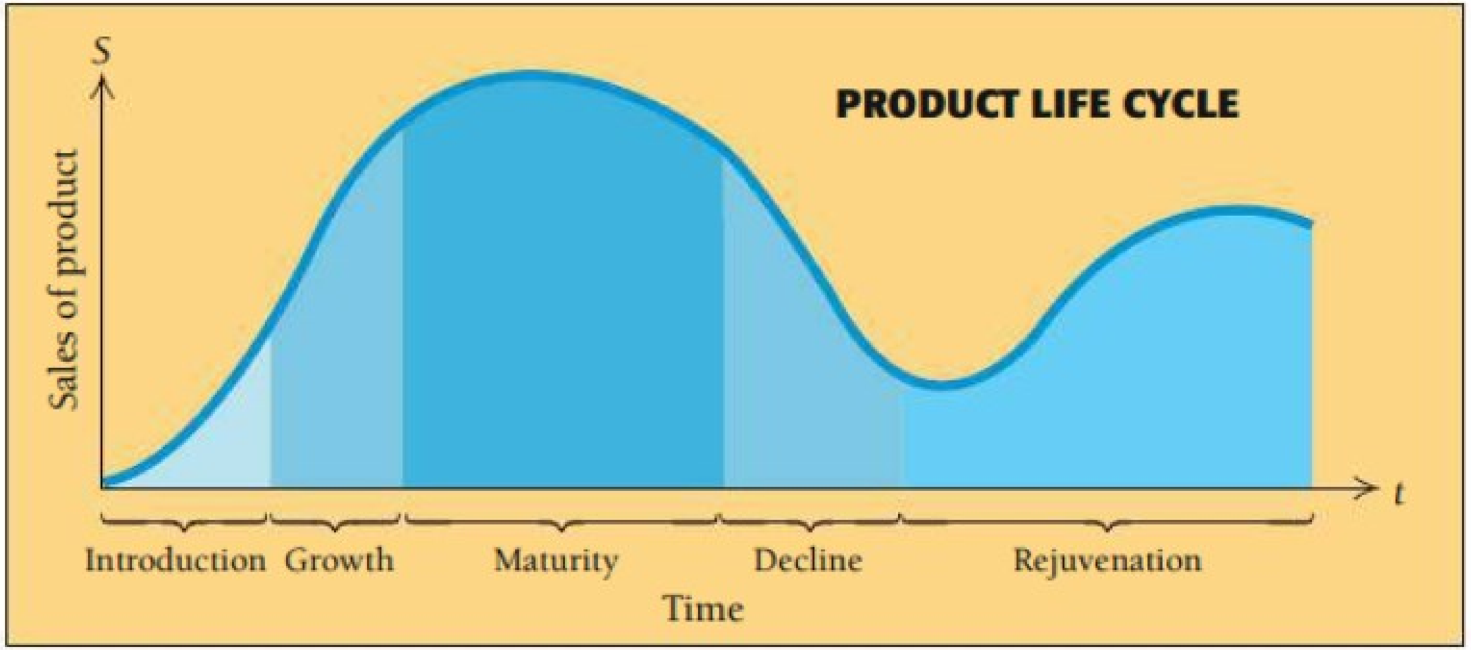
FAR BEYOND

MAT122

Extrema

Stony Brook University

Product Life Cycle



number of items sold
with respect to time

sales start slowly then gradually increase

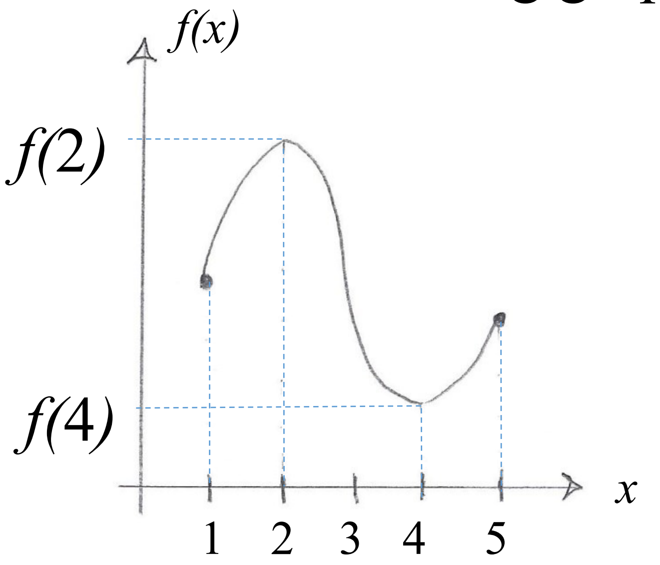
sales peak then decrease

perhaps improvements/upgrades causes a rejuvenation

maximums and minimums can be found using derivatives

Absolute Extrema

consider the following graph:



domain is [1,5] ← a <u>closed</u> interval

<u>consider $\boldsymbol{x = 2}$</u>:
its corresponding y-value is $f(2)$
$f(2)$ is the largest y-value on [1,5]

then an **<u>absolute maximum</u>** occurs at $x = 2$ and its **<u>absolute maximum value</u>** is $f(2)$

collectively, max/min called **<u>extreme values</u>** or **<u>extrema</u>** ← plural

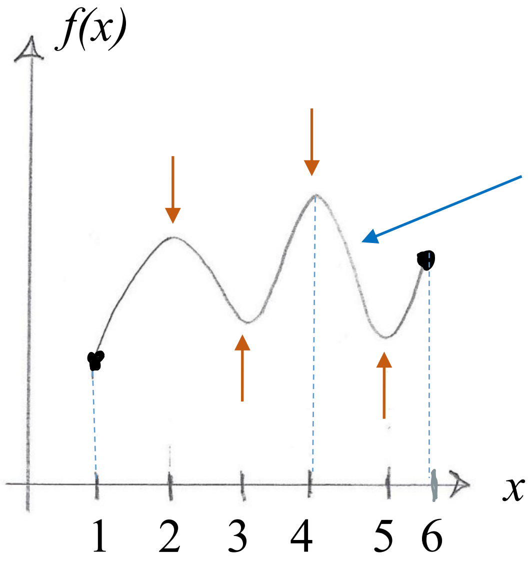
similarly an **<u>absolute minimum</u>** occurs at $x = 4$ and its **<u>absolute minimum value</u>** is $f(4)$

potential max/min called **<u>critical points</u>**

Local Extrema

sometimes there are multiple minima/maxima in a function

plural



also closed interval

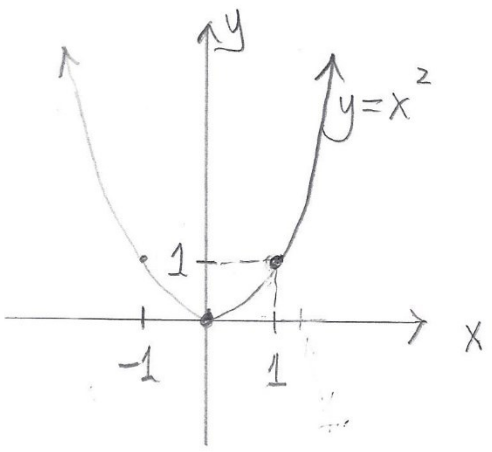
absolute minimum occurs at $x = 1$

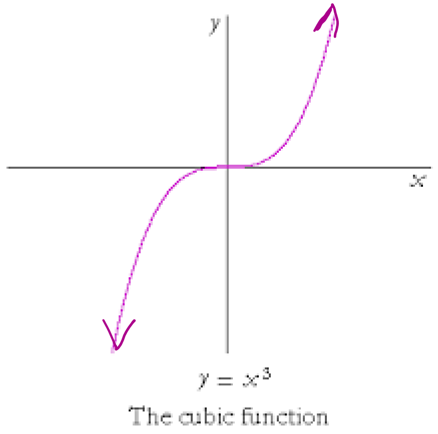
absolute maximum occurs at $x = 4$

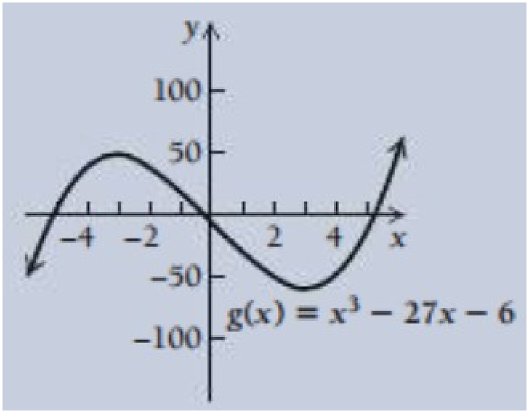
local maxima occur at $x = 2$, $x = 4$

local minima occur at $x = 3$, $x = 5$

Extrema - examples







$y = x^2$

$y = x^3$

$g(x) = x^3 - 27x - 6$

has one minimum
but no maximum

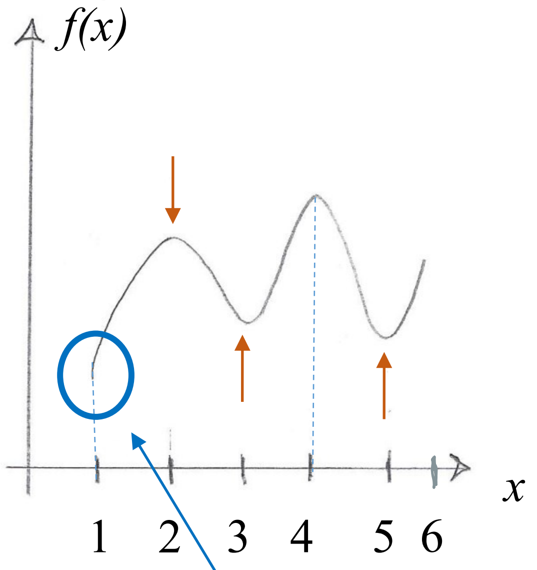
no extrema

has one maximum
and one minimum

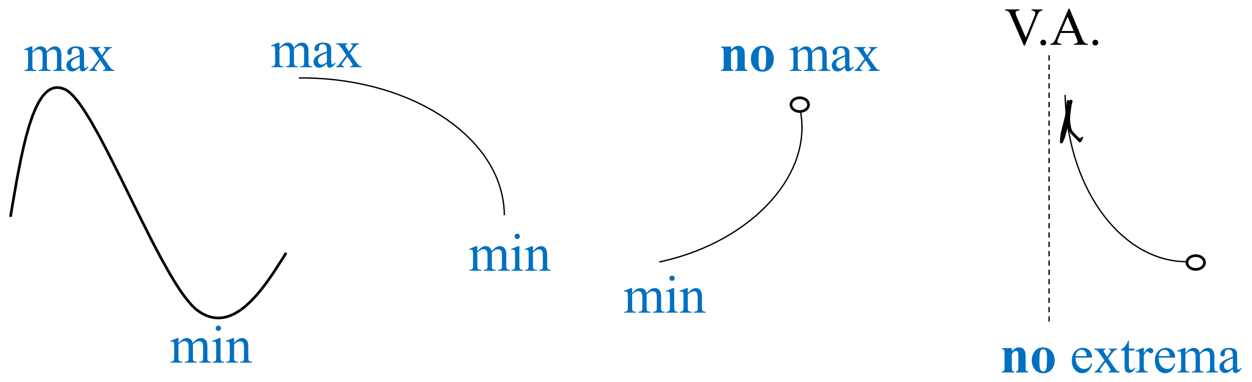
Extrema on a Closed Interval

re-visit previous closed interval graph:

Rule:
If f is continuous on a closed interval, check the endpoints for extrema.

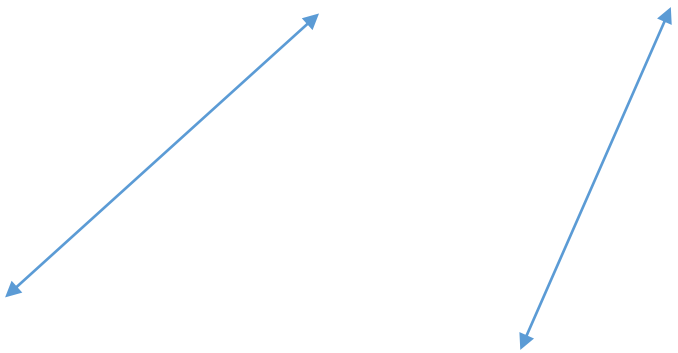


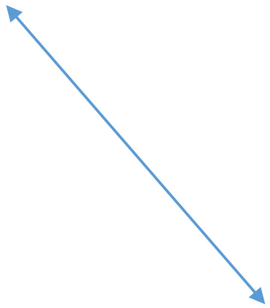
Notice: absolute minimum occurs at an endpoint



Rate of Change - Refresh

Slope measures the **steepness** of a line or the **rate of change** at a place on a curve





steeper slope implies higher rate of change

(*x*-values and *y*-values are both increasing)

negative slope implies negative rate of change

(as *x*-values are increasing, *y*-values are decreasing)

slope is 0

NO change

Meanings of Derivatives – Review #1

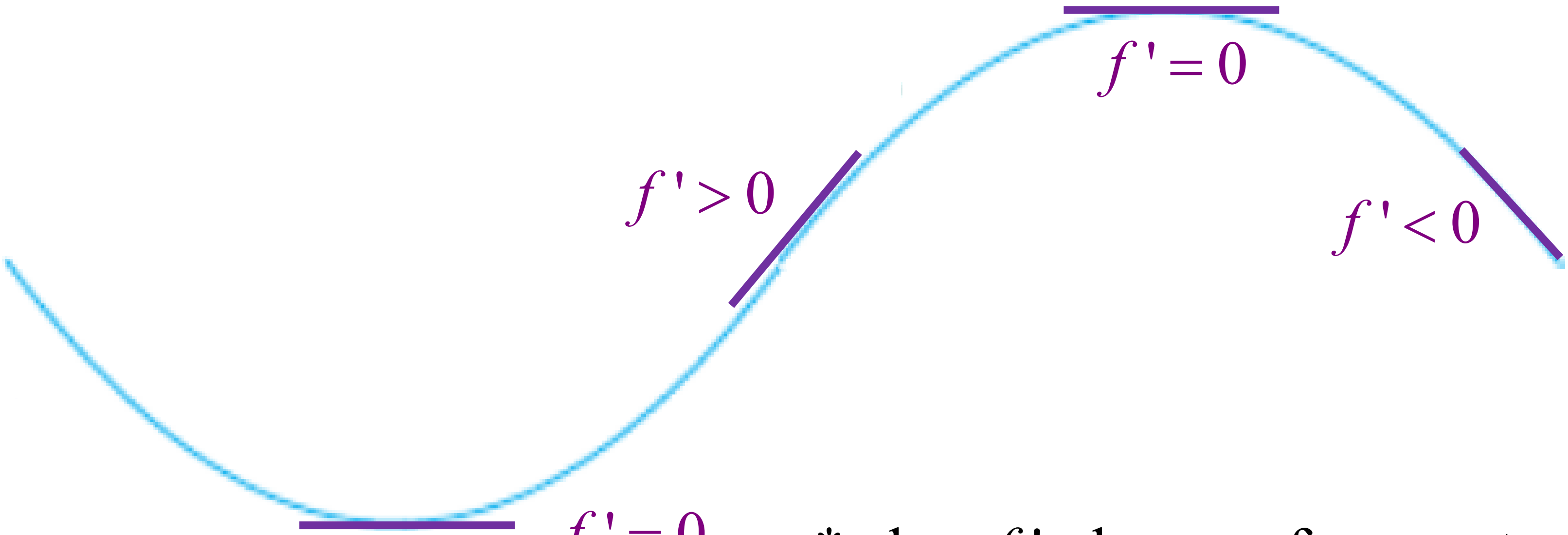
Increasing/Decreasing

If f' > 0 on an interval then f is **increasing** on that interval.

If f' < 0 on an interval then f is **decreasing** on that interval.

when $f' = 0$ f can have a local max or local min *

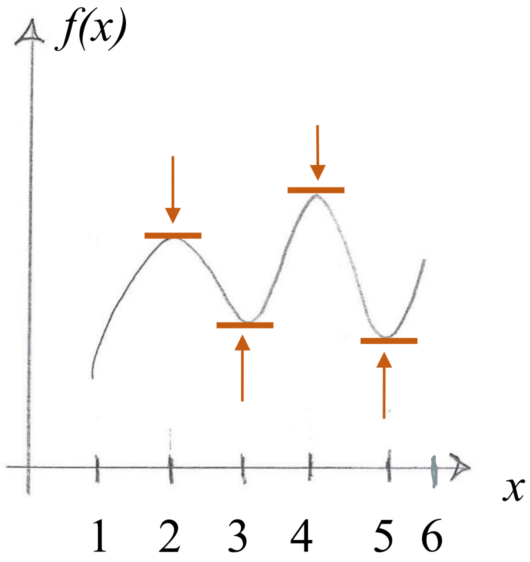
*when f' changes from + to –, → have max



*when f' changes from – to +, → have min

Local Extrema and Slope of Tangent Line

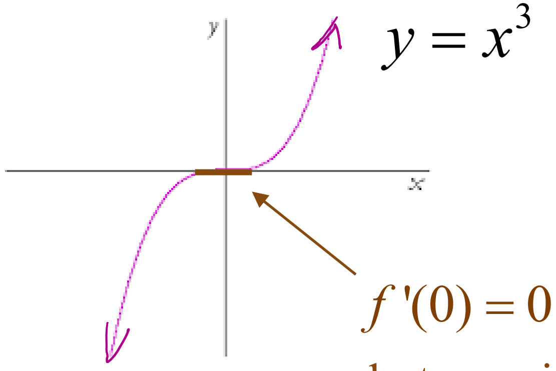
anywhere there is local extrema,
the slope of its tangent line is 0:



Fermat's Theorem:

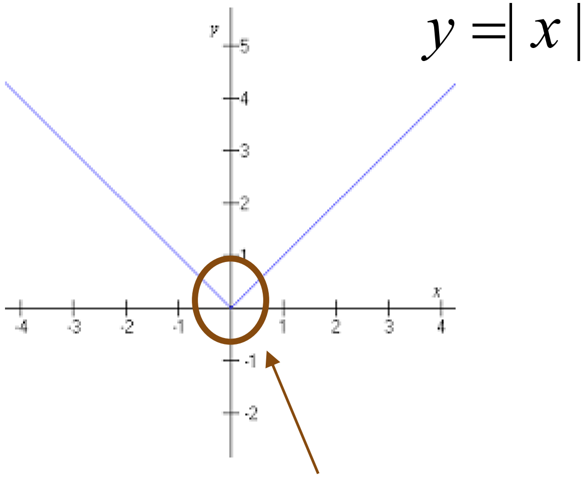
If f has a local max or min at c and $f'(c)$ exists,
then $f'(c) = 0$.

doesn't necessarily work the other way…



$f'(0) = 0$

but no min/max
exist at $x = 0$



min exists at $x = 0$

but $f'(0)$ DNE (corner)

Find Extrema w Differentiation – all reals

ex. Identify extrema of $f(x) = 3x^4 - 16x^3 + 18x^2$

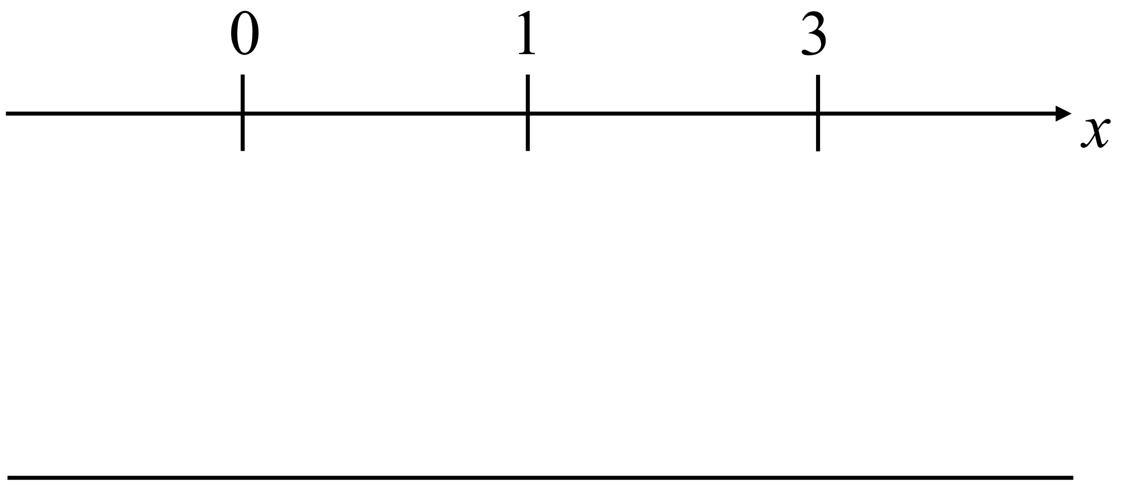
Get critical points by setting first derivative equal to 0.

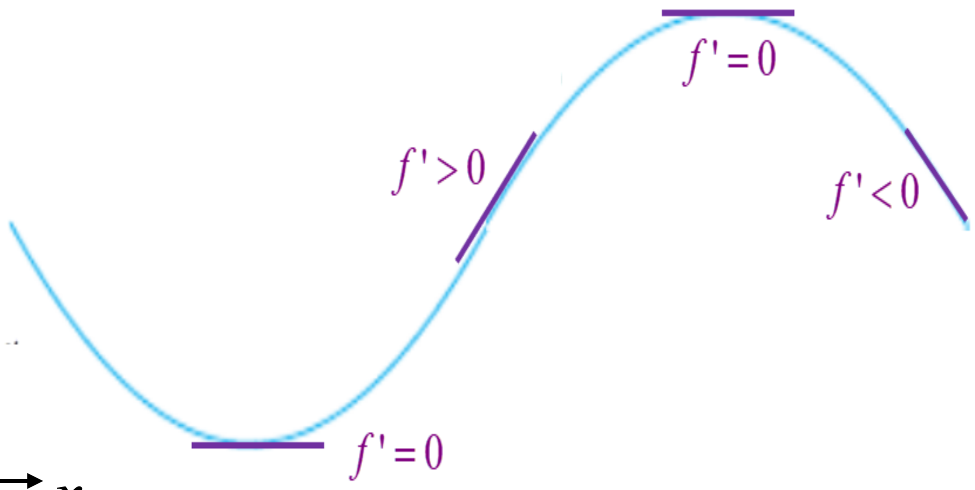
Do: $f'(x) = 12x^3 - 48x^2 + 36x = 0$ factor

$$12x\left(x^2 - 4x + 3\right) = 0$$

$$12x(x-1)(x-3) = 0$$

critical points: $x = 0 \quad x = 1 \quad x = 3$





$12x$

$x - 1$

$x - 3$

$12x(x-1)(x-3)$

Find Extrema – revisit with Closed Interval

ex. Identify extrema of $f(x) = 3x^4 - 16x^3 + 18x^2$ on $-1 \le x \le 4$.

Get critical points by setting first derivative equal to 0.

$$f'(x) = 12x^3 - 48x^2 + 36x = 0$$

$$12x\,(x^2 - 4x + 3) = 0$$

$$12x\,(x-1)(x-3) = 0$$

critical points:

$x = 0$ local min

$x = 1$ local max

$x = 3$ local min

<u>To find **absolute** extrema</u>:

plug **critical points** and **ENDPOINTS** into <u>original</u> function:

$f(0) = 0$

$f(1) = 5$

$f(3) = -27$ **absolute** min

$f(-1) = 37$ **absolute** max

$f(4) = 32$ none

endpoints cannot be LOCAL extrema